

Modélisation non-hydrostatique en
Océanographie
En combinant **frontières virtuelles** et un
modèle **spectral** basé sur les
séries de Fourier

Achim Wirth

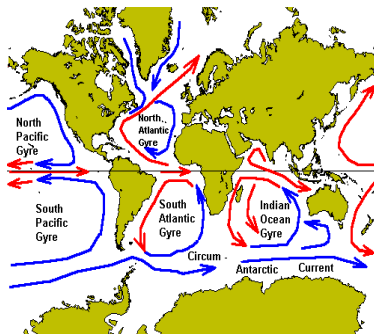
MEIGE / LEGI / CNRS

Atelier Fluide, 18 mars, 2013

Ocean Circulation

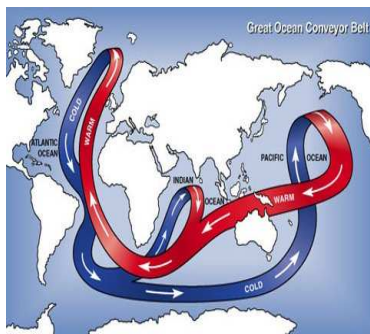
Gyre

“weather”



Overturning

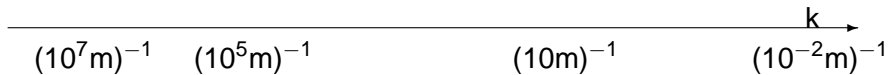
“climat”



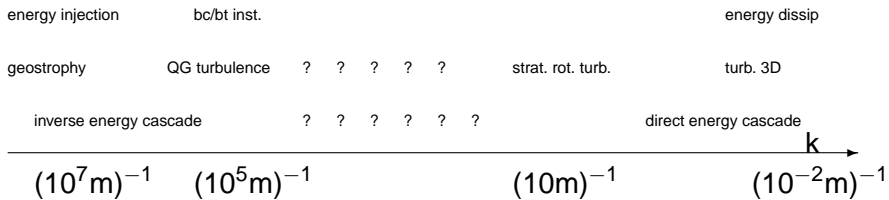
Ocean Dynamics by Scale

energy injection

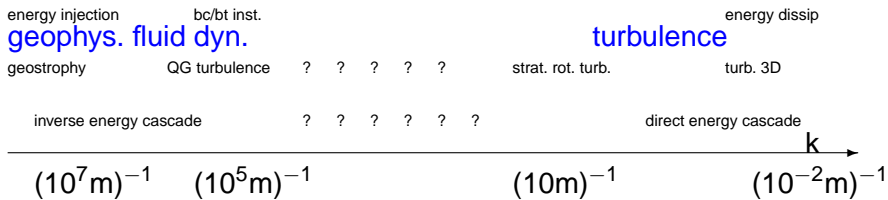
energy dissip



Ocean Dynamics by Scale

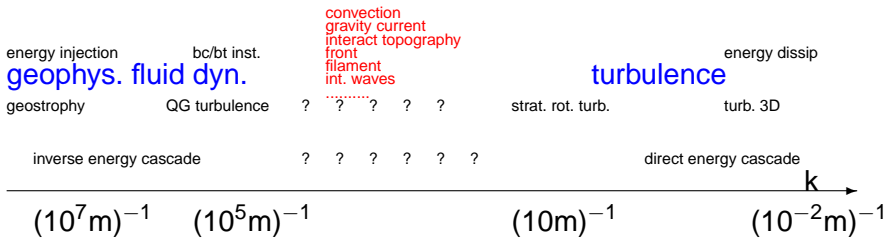


Ocean Dynamics by Scale



Ocean Dynamics by Scale

small scale processes



Phénomènes non-hydrostatiques en Océanographie

- ▶ Convection
- ▶ Courants gravitaires
- ▶ tourbillons océaniques
- ▶ Ondes internes
- ▶ Couche d'Ekman
- ▶ Dynamique de la couche de mélange (Langmuir, ...)
- ▶ Interactions des tourbillons méso-échelles
- ▶
- ▶

Méthode Pseudo-Spectrale

Espace physique

$$u(x) = \sum_k \hat{u}(k) e^{2\pi i k \cdot x}$$

$$\partial_t u = -u^3$$

$$\partial_t u = \nu \nabla^2 u$$

Espace de Fourier

$$\hat{u}(k) = \frac{1}{n} \sum_x u(x) e^{-2\pi i k \cdot x}$$

$$\partial_t \hat{u}(k) = - \sum_{k_1+k_2+k_3=k} \hat{u}(k_1) \hat{u}(k_2) \hat{u}(k_3)$$

$$\partial_t \hat{u}(k) = -\nu k^2 \hat{u}(k)$$

\leftrightarrow

TFR / FFT $O(n \log n)$

Conditions aux Limites et Condition de Divergence Zéro

Espace Physique

Conditions aux limites

$$u(x_B) = 0$$

$$\nabla \cdot u = 0$$

Espace de Fourier

$$\sum_k \beta_k \hat{u}(k) = 0$$

Divergence zero

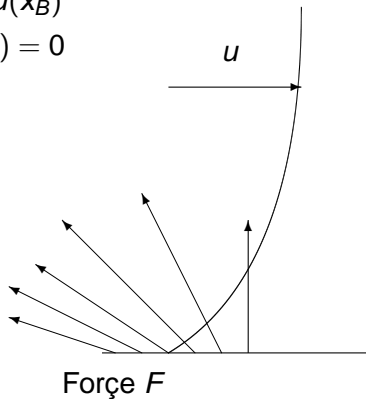
$$\hat{u}(k) = \Pi \left(\hat{\tilde{u}}(k) \right)$$

$$\text{with : } \Pi \left(\hat{\tilde{u}}(k) \right)_i = \hat{\tilde{u}}_i - k^{-2} k_i (k_j \hat{\tilde{u}}_j)$$

Comment imposer les deux conditions simultanément ?

Conditions aux Limites Virtuelles

- ▶ Forçage "Feedback" : $F \mapsto u(x_B)$
- ▶ Forçage direct : $(F \mapsto) u(x_B) = 0$



Le Problème

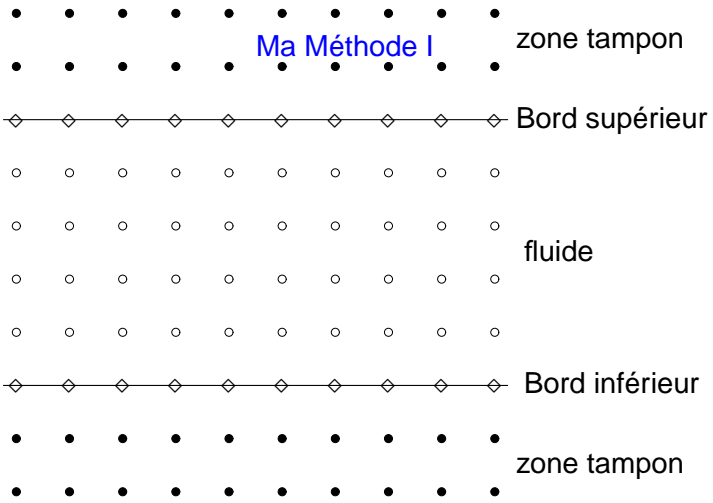
$$\partial_t u + u \cdot \nabla u + 2\Omega \times u + \nabla P = \alpha T \mathbf{e}_\perp + \nu \nabla^2 u + F$$

$$\nabla \cdot u = 0$$

$$\partial_t T + u \cdot \nabla T = \kappa \nabla^2 T + S.$$

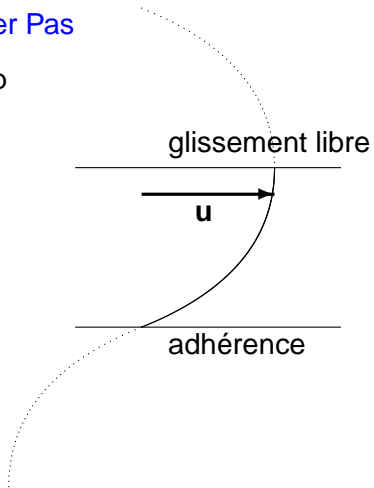
$$u(x_{B1}) = 0$$

$$u(x_{B2}) \cdot \mathbf{e}_\perp = 0$$



Premier Pas

- ▶ Commencer avec un champ de vitesse satisfaisant les conditions aux limites
- ▶ Imposer la symétrie à l'intérieur du « solide »



Deuxième Pas

Intégrer l'équation d'inertie sans tenir compte de la pression
("splitting method")

$$\frac{\tilde{u} - u}{\Delta t} = -u \cdot \nabla u - \alpha T e_{\perp} + \nu \nabla^2 u + F^1.$$

Troisième Pas

Projection :

$$\tilde{u} = \Pi(\tilde{u}),$$

$$\Pi(u) = u - \nabla^{-2} \nabla(\nabla \cdot u),$$

mais

$$\tilde{u}(x_B) \neq 0.$$

(fin pour conditions aux limites virtuelles “classiques”)

Quatrième Pas

Appliquer la force F^2 , telle que $\tilde{u} \rightarrow \tilde{u} + u_\pi$,
et $u = \Pi(\tilde{u} + u_\pi)$ avec

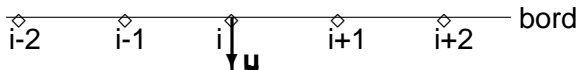
$$u(x_B) = 0.$$

On obtient :

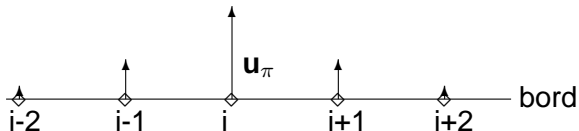
$$\frac{u^{n+1} - u^n}{\Delta t} = \Pi \left(-u^n \cdot \nabla u^n + \alpha T^n + \nu \nabla^2 u^n + F^1 + F^2 \right).$$

Mais comment trouver F^2 ?

Quatrième Pas



Chercher \mathbf{u}_π tel que $\mathbf{u}(\mathbf{x}_B) = -\Pi(\mathbf{u}_\pi)(\mathbf{x}_B)$.



Calcul de u_π

Pb. elliptique de dim. : $n_1 \times n_2$

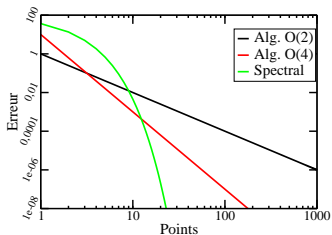
- ▶ Topographie quelconque : Méthode du gradient conjugué.
- ▶ Fond plat : calcul direct en utilisant l'homogénéité.

HARmonic Ocean MODEL

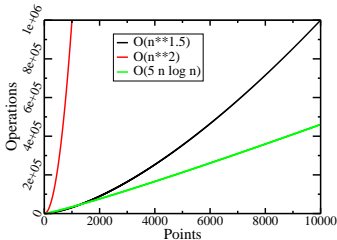
(Wirth, Ocean Modelling 2005)

Méthode Pseudo-Spectrale (Fourier) & Frontière Virtuelle

- convergence spectrale

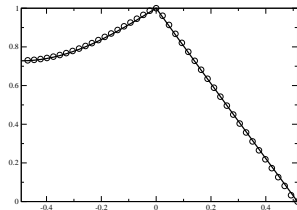
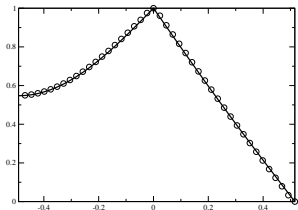
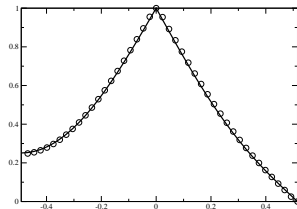
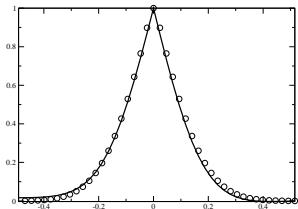


- efficacité spectrale



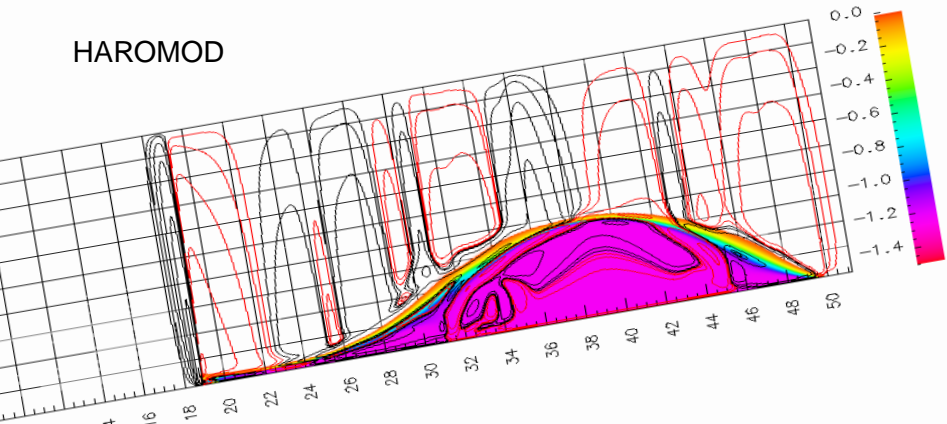
- Parallélisation efficace

Écoulement de Rayleigh Impulsive

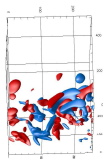
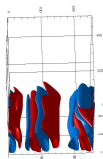
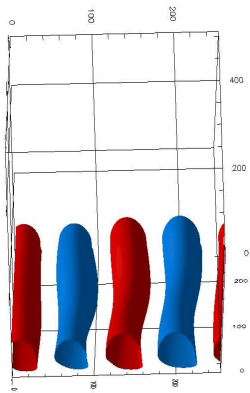


Non-hydrostatic simulation :

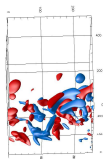
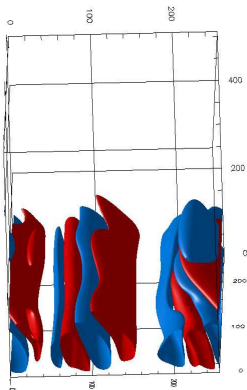
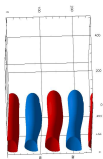
HAROMOD



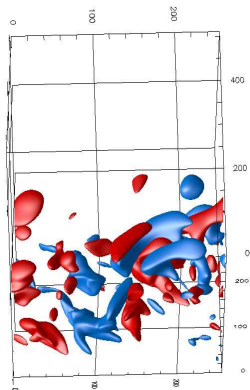
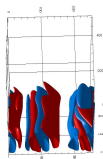
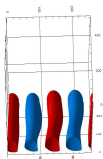
Coherent Structures

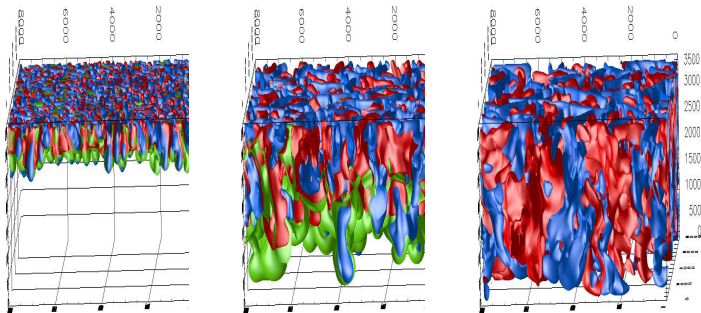


Coherent Structures



Coherent Structures





Bi-periodic domain ($8\text{km} \times 8\text{km} \times 3.5\text{km}$)

Isothermal ocean

Forcing : 250, 500, 1000 W/m^2

Latitude : North pole, Golf of Lions (45°N)

Numerical resolution : $256 \times 256 \times 224$ points

Conclusion

Ca marche

pour les les frontières plates

Conclusion

Ca marche

pour les les frontières planes

Autres applications à voir